

# Quantum Fields and Dissipation

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## Abstract

The description of thermal or non-equilibrium systems necessitates a quantum field theory which differs from the usual approach in two aspects: 1. The Hilbert space is doubled; 2. Stable quasi-particles do not exist in interacting systems. A mini-review of these two aspects is given from a practical viewpoint including two applications. For thermal states it is shown how infrared divergences occurring in perturbative quasi-particle theories are avoided, whereas for non-equilibrium states a memory effect is shown to arise in the thermalization.

## I. INTRODUCTION

When the first decades of our century saw the rapid ascent of quantum theory, physicists were troubled by the question: *How can dissipation and irreversibility arise in the macroscopic nature, although the microscopic equations governing the world are time-reversal invariant?* The only place where dissipation was visible in quantum mechanics at this time was the *decay* of states, but it remained obscure as what initiates such a decay.

Further progress was stalled until the evolution of quantum field theory in the 1940's. It became apparent then, that the vacuum we intuitively see as a void space is indeed a bubbling, fluctuating sea of energy. Among the early contributors to this picture was Hiroomi Umezawa, who realized that this bubbling may couple to the observable world [1]. Since this adds a statistical element to the microscopic world, it became obvious that irreversibility would find its natural explanation in quantum field theory – and people started to think about a connection of field theory to thermodynamics.

It is the purpose of this paper to present a brief review of the modern view on dissipative quantum field theory, which has evolved in the 1990's under Hiroomi Umezawas continuous participation [2].

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The paper is organized as follows: In the next two sections the focus will be on the fundamental aspects of Hilbert space doubling and the breakdown of the quasi-particle picture. Section 4 is devoted to fundamental physical effects in hot plasmas that are connected to finite temperature quantum field theory. In section 5 the picture is extended to non-equilibrium systems, followed by a description of its physical consequences. For these physical examples we chose a toy model of “quarks” in a hot gas of massless bosons, as probably present in relativistic heavy ion-collisions [3] and the early universe [4].

## II. HILBERT SPACE DOUBLING

One of the early findings of finite temperature field theory is the connection between temperature and temporal boundary conditions: The physical excitations propagating into the future are either particles, or they are holes in some sea (like e.g. the vacuum sea). From this follows, that the occupied states of such a sea must propagate backwards in time – a combination, which gives rise to the Feynman boundary condition for vacuum Green functions. For a system at finite, i.e., nonzero temperature, the temporal boundary conditions for Green functions are the Kubo-Martin-Schwinger (KMS) conditions [5].

The KMS conditions take into account the fact, that at finite temperature the states of a system are occupied with a certain probability, hence with this probability are propagating backwards in time. This immediately raises the question, whether one cannot find a new linear combination of particles and holes which eliminates this probabilistic factor from the description.

For non-equilibrium systems this question is certainly harder to answer than for thermal states, because in the latter the occupation probability for each state is fixed. For thermal as well as for non-equilibrium state one therefore carries along the time-forward (retarded) as well as time-backward (advanced) boundary condition. The two-point function of a fermionic quantum field with canonical anti-commutation relation  $\{\psi(t, \mathbf{x}), \psi^\dagger(t, \mathbf{y})\} = \delta^3(\mathbf{x} - \mathbf{y})$  then has an additional  $2 \times 2$  matrix structure. For the implementation of this matrix structure in quantum field theory exist two “flavors”. The oldest is the Schwinger-Keldysh or closed-time path formalism, CTP [6]. However, it has the disadvantage of containing only a single representation of the canonical anti-commutation relation given above.

From the more fundamental viewpoint one is required to have two such representations in a thermal system, which are mutually anti-commuting (commuting for bosons). A proper method to construct these is called thermo field dynamics (TFD), discovered by Hiroumi Umezawa in 1975 [7,8].

In thermo field dynamics (TFD) the field  $\psi$  is complemented by a second field  $\tilde{\psi}_x$  with canonical anti-commutation relations  $\{\tilde{\psi}(t, \mathbf{x}), \tilde{\psi}^\dagger(t, \mathbf{y})\} = \delta^3(\mathbf{x} - \mathbf{y})$  and anti-commuting with  $\psi$ . While  $\psi$  is evolving forward in time,  $\tilde{\psi}$  is subject to a reversed time evolution.

For the purpose of this mini-review we will treat the two methods CTP and TFD as equivalent because the matrix valued propagator of TFD,

$$S^{(ab)}(x, y) = -i \begin{pmatrix} \langle T [\psi_x \bar{\psi}_y] \rangle & -i \langle T [\psi_x \tilde{\psi}_y] \rangle \gamma^0 \\ i\gamma^0 \langle T [\tilde{\tilde{\psi}}_x \bar{\psi}_y] \rangle & -\gamma^0 \langle T [\tilde{\tilde{\psi}}_x \tilde{\psi}_y] \rangle \gamma^0 \end{pmatrix}, \quad (1)$$

is exactly equal to the Schwinger-Keldysh result. For a detailed discussion of this equivalence and the exact meaning of  $\langle \cdot \rangle$  we refer to [2,9]. One virtue of using the closed-time path formalism and TFD together is the easy recognition that *by construction* the above propagators fulfill

$$S^{11}(x, y) + S^{22}(x, y) = S^{12}(x, y) + S^{21}(x, y) \quad (2)$$

for equilibrium as well as for non-equilibrium states.

Actually, for the case of thermal equilibrium, the KMS boundary condition *imposes* the double structure on the Hilbert space of a quantum field theory [10]. For the above propagator, since in equilibrium it depends only on  $(x - y)$ , this KMS condition is most easily expressed after a Fourier transform  $(x - y) \rightarrow P = (p_0, \mathbf{p})$ :

$$(1 - n_F(p_0)) S^{12}(p_0, \mathbf{p}) + n_F(p_0) S^{21}(p_0, \mathbf{p}) = 0, \quad (3)$$

where

$$n_F(E) = \frac{1}{e^{\beta(E-\mu)} + 1} \quad (4)$$

is the Fermi-Dirac distribution function at temperature  $1/\beta$  and chemical potential  $\mu$ . Because there are now two independent linear relationships, (2) and (3), among the matrix elements of the propagator, it is clear that at least in thermal equilibrium it contains a lot of spurious information.

### III. NON-SHELL QUANTUM FIELDS

A second ingredient, labeled the breakdown of the quasi-particle picture, is equally important for quantum field theory of thermal and non-equilibrium states. It is due to the fact, that in general one does not consider systems which at (temporal and spatial) infinity consist of free particles. Rather, the physical systems we are interested in lack such a free asymptotic condition and need to be described with a more general asymptotic condition [11,12]. To express this formally, perform the Fourier transform of the propagator with respect to the difference  $(x - y)$  also in non-equilibrium states, which then defines a mixed (Wigner) representation in terms of the momentum variable  $P = (p_0, \mathbf{p})$  and  $X = (x + y)/2$ .

In this representation, the retarded and advanced propagator are determined by a dispersion integral

$$\begin{aligned} S^{R,A}(X; p_0, \mathbf{p}) &\equiv S_{XP}^{R,A} = G_{XP} \mp i\pi \mathcal{A}_{XP} \\ &= \int_{-\infty}^{\infty} dE \frac{\mathcal{A}(X; E, \mathbf{p})}{p_0 - E \pm i\epsilon} \end{aligned} \quad (5)$$

over a generalized spectral function  $\mathcal{A}_{XP}$ , i.e., retarded and advanced propagator have a common analytical continuation away from the real energy axis. For the free case this function would be  $\mathcal{A}(E, \mathbf{p}) \rightarrow (E\gamma^0 + \mathbf{p}\boldsymbol{\gamma} + m) \text{sign}(E) \delta(E^2 - \mathbf{p}^2 - m^2)$ . The properties of the "spectral" function in thermal and non-equilibrium states follow from the absence

of the free asymptotic condition: For thermal systems it has been rigorously proven [13], that  $\mathcal{A}$  must not contain isolated poles on the real energy axis, i.e., quasi-particles do not exist at finite temperature. In other words, the irreducible representations of the space-time symmetry group at finite temperature do not have a mass shell. Since the dispersion integral furthermore shifts *complex* poles onto the unphysical Riemann sheet, the only non-analyticity for a thermal or non-equilibrium propagator therefore are cuts along the real energy axis.

In the thermal equilibrium case, the mixed representation has no  $X$ -dependence. One may therefore use the KMS condition and the linear relation (2) together with the definition of retarded and advanced propagator to obtain the matrix valued propagator in equilibrium as [2,9]

$$S^{(ab)}(p_0, \mathbf{p}) = \int_{-\infty}^{\infty} dE \mathcal{A}(E, \mathbf{p}) \times \tau_3 (\mathcal{B}(n_F(E)))^{-1} \begin{pmatrix} \frac{1}{p_0 - E + i\epsilon} & \\ & \frac{1}{p_0 - E - i\epsilon} \end{pmatrix} \mathcal{B}(n_F(E)) \quad (6)$$

with a  $2 \times 2$  Bogoliubov matrix  $\mathcal{B}$  depending on a single parameter  $n$  as

$$\mathcal{B}(n) = \begin{pmatrix} (1-n) & -n \\ 1 & 1 \end{pmatrix}. \quad (7)$$

Diagram rules, transport theory and other developments of traditional thermal field theory may be carried over to this treatment in terms of spectral functions [9].

For the purpose of the present paper it is sufficient to restrict the discussion to a simplified ansatz for such a spectral function, which contains two independent parameters: A dynamical mass  $m$  and a spectral width  $\gamma$ , both are for the sake of a simple approximation taken as energy and momentum independent

$$\mathcal{A}(E, \mathbf{p}) = \frac{\gamma \gamma^0 (E^2 + \omega^2 + \gamma^2) + 2E\boldsymbol{\gamma}\mathbf{p} + 2Em}{\pi (E^2 - \omega^2 - \gamma^2)^2 + 4E^2\gamma^2}, \quad (8)$$

with  $\omega^2 = m^2 + \mathbf{p}^2$ . One may regard such a spectral function as the generalization of the standard  $\delta$ -function energy-momentum relation to a broader distribution for thermally scattered particles, e.g. to a kind of Lorentzian curve.

#### IV. A PHYSICAL EFFECT IN THERMAL SYSTEMS

In the following a physical effect is discussed which arises from the breakdown of the quasi-particle picture in finite temperature field theory: If one performs a quasi-particle calculation of the photon radiation rate out of a hot plasma, it diverges in the infrared (soft photon) sector [14]. It will be shown, how the introduction of a nontrivial fermion spectral function cures this problem. For brevity, we present only a few results, and refer to [15] for the technical details.

In a hot equilibrated plasma, the Hilbert space doubling has to be carried out also for the photons, i.e., photon self energies as well as propagators exhibit the  $2 \times 2$  matrix structure also found above. The *radiation rate* of these photons, i.e., the emission rate through an artificial boundary put into the system is given by the unordered (Wightman, or 12-) matrix element of the photon self energy  $\Pi$ .

Apart from the  $2 \times 2$  matrix structure due to the Hilbert space doubling however, this self energy function also is a tensor in Minkowski space. A gauge invariant photon production rate is obtained in the sum over all polarizations,  $\epsilon_\mu \epsilon_\nu \Pi^{\mu\nu} = \Pi^\mu_\mu$ .

The question of gauge invariance requires a careful discussion, because usually it necessitates a calculation of  $\Pi$  in two-loop order (otherwise  $\Pi$  will violate current conservation). However, with our ansatz spectral function we are on a safe side: The *necessary* vertex correction drops out of this sum over polarizations, and one may obtain a gauge invariant photon radiation rate

$$R(E_\gamma, T) = E_\gamma \frac{dN_\gamma}{d^3\mathbf{p}} = 2 \frac{n_B(E_\gamma, T)}{8\pi^3} \text{Im} (\Pi_{11}^R + \Pi_{22}^R) = \frac{i}{8\pi^3} (\Pi_{11}^{12} + \Pi_{22}^{12}) . \quad (9)$$

Here, the lower indices are Lorentz indices, and the upper indices refer to the  $2 \times 2$  matrix structure of the Hilbert space doubling.  $n_B$  is the Bose-Einstein distribution function for the photons, which appears when exploiting the KMS condition relating the 12 matrix element of  $\Pi$  to the imaginary part of the retarded polarization function. The latter we take from ref. [9] as

$$\begin{aligned} \text{Im} \Pi_{\mu\nu}^R(k_0, \mathbf{k}) &= -\pi e^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \int_{-\infty}^{\infty} dE \\ &\quad \text{Tr} [\gamma_\mu \mathcal{A}(E + k_0, \mathbf{p} + \mathbf{k}) \gamma_\nu \mathcal{A}(E, \mathbf{p})] (n_F(E) - n_F(E + k_0)) . \end{aligned} \quad (10)$$

where  $e$  is the electric charge of the fermion and  $\mathcal{A}$  is the spectral function including the spectral width parameter  $\gamma$ .

In the next step, a model for the mass and spectral width of the fermions is needed. In ref. [15] it is discussed, how they may be obtained for “quarks” in a strongly coupled Nambu–Jona-Lasinio model. In this model one obtains a strongly temperature dependent “quark” mass and spectral width due to a chiral phase transition – and consequently also a very interesting temperature dependence of the photon radiation rate. Since the present paper is devoted to the fundamentals of dissipative field theory, this model is not discussed in greater detail.

Instead, we have simply plotted the result for the photon radiation rate for a degenerate plasma of constituent “quarks” with 300 MeV mass, and a purely electromagnetic spectral width  $\gamma^{\text{em}}$ . Although this width  $\gamma^{\text{em}}$  is, in principle, a non-analytical function of the temperature, the smallness of the electromagnetic coupling constant  $\alpha = e^2/4\pi$  allows to approximate it very well by the lowest order result

$$\gamma^{\text{em}}(T) \approx \frac{5}{9} \alpha T , \quad (11)$$

where the factor  $5/9$  is due to the (u,d)-family averaging of the “quark” electric charge.

In figure 1 it is shown, that this calculation leads to a photon production rate which for high photon energies is suppressed by a Boltzmann factor – whereas it saturates for photon energies  $E_\gamma \leq 2\gamma^{\text{em}}$  and does *not* lead to an infrared divergence.

Thus it is found, that when properly taking into account the breakdown of the quasi-particle picture, i.e., when substituting a spectral function that does not exhibit isolated poles, the infrared problem is solved.

One may conclude the discussion of this equilibrium physical effect by relating it to an intuitive picture: The spectral broadening of fermions in a hot plasma is due to their repeated thermal scattering, i.e., to a kind of Brownian motion. The average distance between two such scattering events is  $\propto 1/(2\gamma)$ , and consequently the fermion forms an "antenna" for electromagnetic radiation which is not longer than this distance. This leads to the cutoff of the soft photon radiation.

## V. RELAXATION IN NON-EQUILIBRIUM SYSTEMS

Having outlined a physical effect of the spectral broadening in thermal systems, we now turn to the question of dissipation in non-equilibrium systems. The results discussed above already indicate, that the spectral function parameter  $\gamma$ , i.e., the spectral width of the "particle", is due to collisions present in a system. Consequently one may expect this parameter also to be associated to relaxation processes in non-equilibrium systems.

Such relaxation processes are usually described by transport equations – and hence the task for the present paper is to give an overview of the connection between those transport equations on one side and dissipative field theory on the other side. To this end, one has to study the Schwinger-Dyson equation for the full fermion propagator in coordinate space  $S = S_0 + S_0 \odot \Sigma \odot S$ , where  $S_0$  is the free and  $S$  the full two-point Green function of the fermion field,  $\Sigma$  is the full self energy and the generalized product  $\odot$  is a matrix product (thermal and spinor indices) and an integration (each of the matrices is a function of two space coordinates). When switching to the mixed (Wigner-) representation as introduced above, one has to perform a nontrivial step to handle the convolution integrals. Formally, their Wigner transform may be expressed as a gradient expansion

$$\int d^4(x-y) \exp(iP_\mu(x-y)^\mu) \Sigma_{xz} \odot G_{zy} = \exp(-i\Diamond) \tilde{\Sigma}_{XP} \tilde{G}_{XP} . \quad (12)$$

$\Diamond$  is a 2nd order differential operator acting on both functions behind it like a Poisson bracket  $\Diamond A_{XP} B_{XP} = \frac{1}{2}(\partial_X A_{XP} \partial_P B_{XP} - \partial_P A_{XP} \partial_X B_{XP})$ . Henceforth will be used the infinite-order differential operator  $\exp(-i\Diamond) = \cos \Diamond - i \sin \Diamond$ . Similar to the propagator in eq. (5), the self energy is split into real Dirac matrix valued functions  $\Sigma_{XP}^{R,A} = \text{Re}\Sigma_{XP} \mp i\pi\Gamma_{XP}$ . One then inserts these expressions into the Schwinger-Dyson equation, and performs a careful split into real and imaginary parts [9]. The resulting diagonal components of the matrix valued Schwinger-Dyson equation are

$$\begin{aligned} \text{Tr}[(P^\mu \gamma_\mu - m) \mathcal{A}_{XP}] &= \cos \Diamond \text{Tr}[\text{Re}\Sigma_{XP} \mathcal{A}_{XP} + \Gamma_{XP} G_{XP}] \\ \text{Tr}[(P^\mu \gamma_\mu - m) G_{XP}] &= \text{Tr}[1] + \cos \Diamond \text{Tr}[\text{Re}\Sigma_{XP} G_{XP} - \pi^2 \Gamma_{XP} \mathcal{A}_{XP}] , \end{aligned} \quad (13)$$

i.e., a closed set of differential equations. Two important facts about these equations have to be emphasized. First notice that these equations do not in general admit a  $\delta$ -function solution for  $\mathcal{A}_{XP}$  even in zero order of  $\diamond$ .

Secondly, the equations do not contain odd powers of the differential operator  $\diamond$ . This implies, that when truncating the Schwinger-Dyson equation to first order in  $\diamond$  (the usual order for the approximations leading to *kinetic* equations), the spectral function  $\mathcal{A}_{XP}$  may still be obtained as the solution of an algebraic equation.

The original Schwinger-Dyson equation was matrix valued. Although eq. (2) implies, that one of the components is spurious, a third differential equation still remains to be solved. It may be transformed into

$$\begin{aligned} \text{Tr} \left[ (\partial_X^\mu \gamma_\mu + 2 \sin \diamond \text{Re} \Sigma_{XP} + \cos \diamond 2\pi \Gamma_{XP}) S_{XP}^K \right] = \\ 2i \text{Tr} \left[ i \sin \diamond \Sigma_{XP}^K G_{XP} - \cos \diamond \Sigma_{XP}^K i\pi \mathcal{A}_{XP} \right] . \end{aligned} \quad (14)$$

with  $S^K = (S^{12} + S^{21})/2$  and  $\Sigma^K = (\Sigma^{12} + \Sigma^{21})/2$ . Note, that here even as well as odd powers of the operator  $\diamond$  occur, and the solution in zero order  $\diamond$  is not trivial. To see this more clearly, we *define* the generalized covariant distribution function  $N_{XP}$  through the equation

$$(1 - N_{XP}) S_{XP}^{12} + N_{XP} S_{XP}^{21} = 0 , \quad (15)$$

and then find that eq. (14) is a differential equation for  $N_{XP}$ . Consequently, this third "off-diagonal" equation is the *transport equation* giving us the desired connection to classical physics.

Before drawing some general conclusions, an application of the formalism developed here will be discussed briefly. The following toy model is used: A gas of bosons (gluons) is instantaneously heated to a very high temperature. In this gas then eventually "quark-antiquark" pairs start to pop up, until at the very end a thermal equilibrium is reached. If these "gluons" dominate the self energy function for the few "quarks" in the medium, and if the back-reaction of quarks on the gluons may be neglected, this amounts to an imaginary part of the self energy function as

$$\Gamma_{XP} \equiv \Gamma_t = \gamma^0 g T(t) = \gamma^0 g (T_i \Theta(-t) + T_f \Theta(t)) \quad (16)$$

Furthermore, one may neglect the influence of anti-quarks in the spectral function (8), but now the dynamical mass (and therefore  $\omega \equiv \omega_t$ ) and spectral width  $\gamma \equiv \gamma_t$  are time-dependent.

With this spectral function, the coupled system (13) reduces to a *single* nonlinear equation for  $\gamma_t$  plus the condition  $\omega_t^2 = \omega_0^2 = \mathbf{p}^2 + m^2$ . This latter condition is more complicated, when the anti-particle piece of the spectral function is taken into account. The energy parameter is chosen as  $E = \omega_0$ , which yields instead of eq. (13) as the Schwinger-Dyson equation for the retarded (or advanced) two-point function of the quarks:

$$\gamma_t = g T_i + g (T_f - T_i) \Theta(t) \left( 1 - e^{-2\gamma_t t} \right) \quad (17)$$

In fig. 2, the solution of this equations is plotted in comparison to the time dependent imaginary part of the self energy function from eq. (16). It is obvious, that the solution of

the nonlinear equation (17) approaches the imaginary part of the self energy function with a characteristic delay time.

Now consider two different levels of transport theory for this model, the corresponding generalized distribution functions are labeled  $N_t$  and  $N_t^B$ . First of all, due to the simplicity of our spectral and self energy function ansatz, the full quantum transport equation (14) reduces to

$$\frac{d}{dt}N_t = -2\gamma_t(N_t - n_F(m, T(t))) \quad (18)$$

with  $T(t)$  as defined in eq. (16). Indeed it turns out, that the spectral width parameter is responsible for the irreversible time evolution of the non-equilibrium state.

This equation looks surprisingly similar to a kinetic equation in relaxation time approach. However, this similarity is superficial: The *kinetic* equation, or Boltzmann equation, derived for our simple model system reads

$$\frac{d}{dt}N_t^B = -2\Gamma_t(N_t^B - n_F(m, T(t))) \quad (19)$$

The difference between the two transport equations in this simple toy model is therefore the occurrence of the self-consistent spectral width parameter in the quantum transport equation, whereas the Boltzmann relaxation parameter is given by the imaginary part of the self energy.

In fig.3 the numerical solution for  $N_t$  is shown and compared to the Boltzmann solution  $N_t^B$ . The comparison of the two methods shows, that the full quantum transport equation results in a *much* slower equilibration process than the Boltzmann equation.

This result is in agreement with other attempts to solve the quantum relaxation problem: The quantum system exhibits a memory, it behaves in an essentially non-Markovian way. The reason for this behaviour is the time-dependent spectral width, which follows the imaginary part of the self energy function only with some delay time.

In particular, for the physical scenario studied here, the time to reach  $1-1/e^2 \approx 86\%$  of the equilibrium “quark” occupation number is almost doubled (14.7 fm/c as compared to 8.2 fm/c in the Boltzmann case).

Thus, although we only used a toy model, it might turn out that quantum effects (= memory as described in this contribution) substantially hinder the thermalization of a strongly interacting plasma over long time scales. A more thorough discussion of this physical result is carried out in ref. [16].

## VI. CONCLUSION

In the preceding sections it was shown, how the two principal features of dissipative field theory, i.e., *1. Hilbert space doubling* and *2. Continuous spectral functions* provide a unified description of equilibrium and non-equilibrium phenomena in statistical systems.

In particular, the spectral width parameter provides the regularization of unphysical divergences as well as the relaxation rate for non-equilibrium states; it may be considered a system parameter as important as the dynamical mass. Of course, the toy model examples



we have considered here are much too simple for deeper physical conclusions – in a realistic system, the energy-momentum dependence of the spectral function is certainly not negligible.

It is also obvious by now, that indeed the dissipation we experience in the macroscopic world has its expression already on the microscopic level: In quantum field theory of thermal systems, each state has an infinite number of infinitely close neighbor states. Thus, while for any finite number or even countably infinite number of degrees of freedom the time evolution of a system will be reversible, this is no longer the case in quantum field theory. Continuous spectral functions, as fundamental feature of thermal systems, will always exhibit irreversible behaviour.

Let us as the final part of this paper add a comment on the connection of this unified view with the work of Hiroomi Umezawa. As seen above, the equilibrium propagator (6) admits a *diagonalization* with a Bogoliubov transformation matrix  $\mathcal{B}$ . This concept of a thermal Bogoliubov transformation was first introduced by Umezawa and collaborators [7], and finally has led to the diagonalization transformation [2]. One may now close the arc of the argumentation by realizing that the proper *non-equilibrium* solution  $N_{XP}$  of the generalized transport equation (14) allows for a *diagonalization* also in general states [9]:

$$\mathcal{B}(N_{XP}) \tau_3 S_{XP} (\mathcal{B}(N_{XP}))^{-1} = \begin{pmatrix} G_{XP} - i\pi \mathcal{A}_{XP} & \\ & G_{XP} + i\pi \mathcal{A}_{XP} \end{pmatrix}. \quad (20)$$

The physical and mathematical understanding of this Bogoliubov symmetry has been achieved in thermo field dynamics. The *diagonalization condition* in equilibrium states is equivalent to the KMS condition, i.e., it specifies the *diagonalization parameter* unambiguously as a Bose-Einstein or Fermi-Dirac distribution function. In non-equilibrium states, the parameter is obtained as the solution of a transport equation [2].

This unified view therefore may be seen as the final achievement of Hiroomi Umezawa, and I am very grateful to the Creator of all worlds that I had the opportunity to share this achievement with Dr. Umezawa.

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# FIGURES

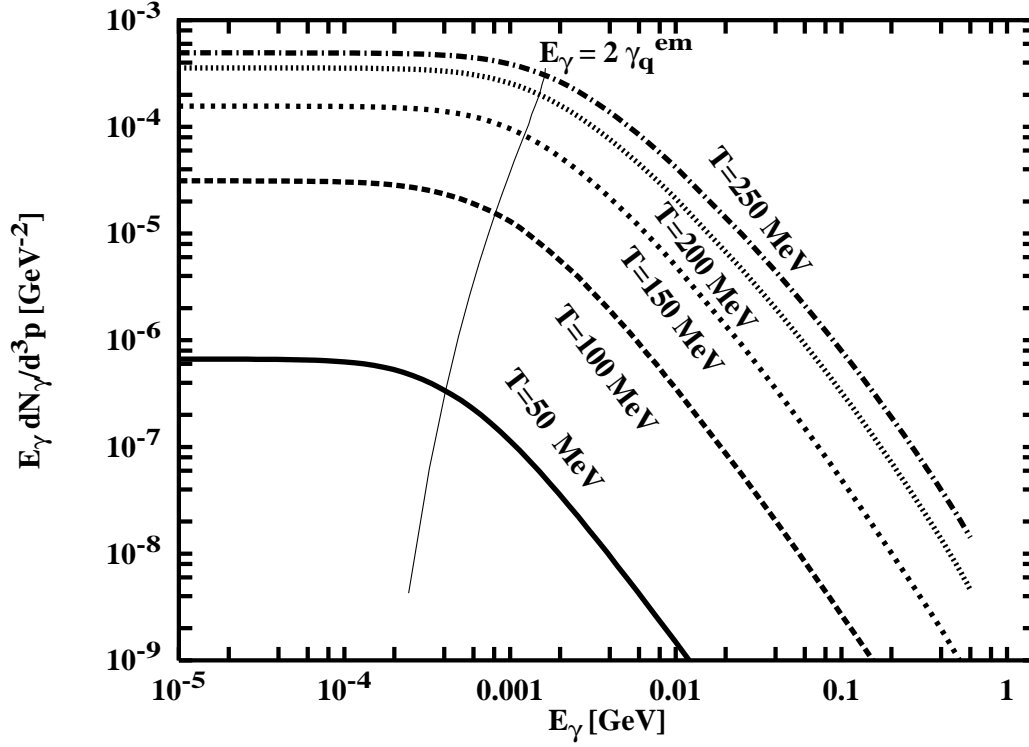


FIG. 1. Photon production rate  $R_\gamma$  from an electromagnetically interacting particle of 300 MeV mass in a plasma as a function of the photon energy  $E_\gamma$  for different temperatures  $T$ .

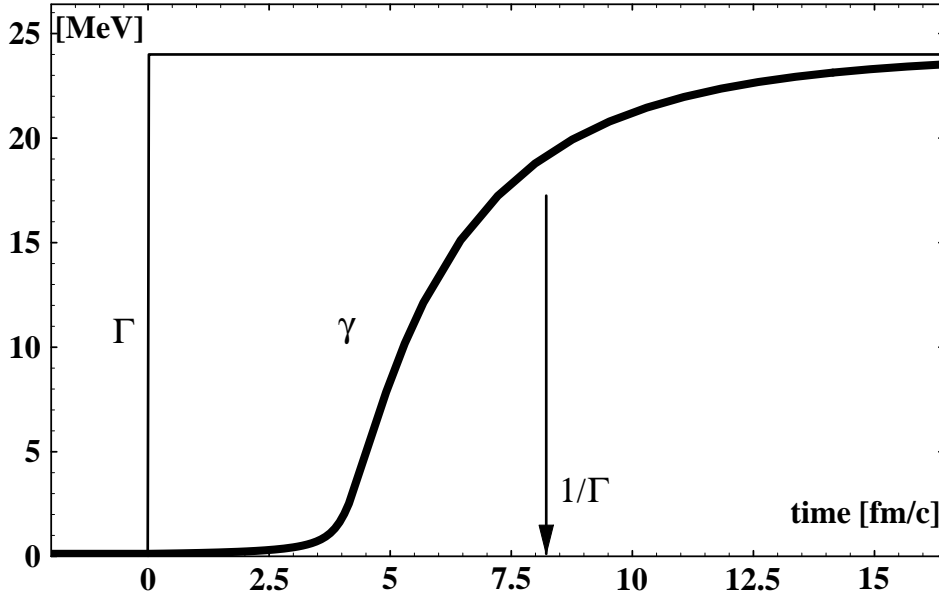


FIG. 2. Time dependent spectral width parameter  $\gamma_t$ . Parameters are  $g=0.12$ ,  $T_i = 1$  MeV,  $T_f = 200$  MeV,  $m = 10$  MeV. Thin line:  $\Gamma_t$  from eq. (16), thick line:  $\gamma_t$  from eq. (17).

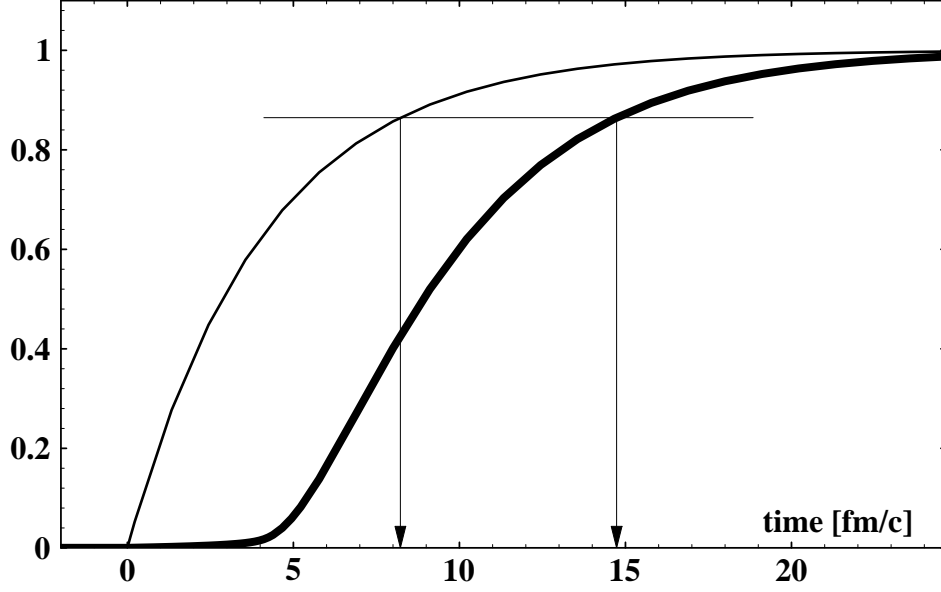


FIG. 3. Normalized time dependent fermionic distribution function for slow quarks. Parameters as in Fig. 2; thin line  $N_t^B/n_F(m, T_f)$  from the Boltzmann equation (19), thick line  $N_t/n_F(m, T_f)$  from the quantum transport equation (18);

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